Grade 4 Mathematics, Quarter 4, Unit 4.1

Applying Knowledge of Measurement to Make Conversions

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Know relative sizes of measurement units within one system (km, m, cm, kg, g, lb, oz, l, ml, hour, minute, second).
- Convert measurements from a larger unit to a smaller unit.
- Record measurement equivalents in a two column table.
- Solve word problems using four operations involving distances, intervals of time, liquid volumes, and masses of objects.
- Represent measurement quantities using diagrams (eg. number line diagram).

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities in problem situations.
- Consider units involved.
- Attend to the meaning of quantities.

Attend to precision.

- Label for clarification.
- Define mathematical symbols and units of measure consistently and appropriately.

Look for and express regularity in repeated reasoning.

- Look for repeated calculations/patterns.
- Accurate calculations.

Essential questions

- In customary units, how do you know what units to use when measuring length, capacity, and weight?
- In metric units, how do you know what units to use when measuring length, capacity, and weight?
- When would you need to convert measurements within a system, and how would you do it?
- How do you represent measurement data using a line plot?
- How do you solve problems involving intervals of time?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, ~~and money, including problems involving simple fractions or decimals,~~ and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

By the end of second grade, students had learned to measure and estimate lengths in inches, feet, centimeters and meters. Additionally, students told time from an analog and digital clocks to the nearest 5 minutes.

Students in grade 3 learned to tell time to the nearest minute and measure time intervals in minutes. They also solved word problems by adding and subtracting minutes. In addition, students represented their solution using a model. They measured and estimated liquid volumes (l) and masses (g and kg)

Word problems involved adding, subtracting, multiplying and dividing masses and volumes given the same units. Additionally, students measured length to the nearest 1/2 inch and 1/4 inch.

Current Learning

Fourth grade students use their understanding of m, cm, kg, g, l, hr, and min to solve word problems. They develop an understanding of km, lb, oz, ml, and seconds and apply this understanding to word problems. Fourth graders convert larger units to smaller units and record on two column table. Use the four operations to solve word problems involving distance, intervals of time, liquid volumes, masses of objects and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement qualities using diagrams such as a number line.

Future Learning

In grade 5, students will convert like measurement units within a given measurement system. They will convert among different-sized standard measurement units within a given system. They will use these conversions to solve multi-step real world problems.

Additional Findings

Principles and Standards for School Mathematics emphasizes that an expanded number and range of measurement tools and techniques should be available to students at this level. When using conventional tools, such as rulers and tape measures for measuring conventional length, students will need instruction to learn to use these tools properly (p. 173).

The work that students do with units of measure and with multiplication of a whole number can be connected to the idea of "times as much" in multiplication. (*PARCC Model Content Frameworks for Mathematics*, p. 20)

As the name suggests, measurement data comes from taking measurements. For example, if every child in a class measures the length of his or her hand to the nearest centimeter, then a set of measurement data is obtained. Other ways to generate measurement data might include measuring liquid volumes with graduated cylinders or measuring room temperatures with a thermometer. In each case, the Standards call for students to represent measurement data with a line plot. (*Progressions for the Common Core State Standards in Mathematics: Grades 2-5, Measurement Data,* p. 2)

Grade 4 Mathematics, Quarter 4, Unit 4.2

Interpreting and Representing Data with Fractions

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Create line plots using fractional measurements (1/2, 1/4, 1/8).
- Analyze data from a line plot to formulate conclusions or solve problems.
- Solve problems involving addition and subtraction of fractions by using information from line plots.

Mathematical practices to be integrated

Model with mathematics.

- Use visual models to represent the problem**.**
- Identify important quantities and express them as an equation**.**

Use appropriate tools strategically.

- Use grade level appropriate tools.
- Decide when tools are appropriate and helpful.

Look for and make use of structure.

- Look for patterns.
- Break numbers apart.

Essential questions

- How can you use a line plot to represent data?
- How can a line plot be helpful in solving problems?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data 4.MD

Represent and interpret data.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection*.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students in grade 3 drew scaled picture graphs and bar graphs to represent data. They solved one-step and two-step problems using information from a bar graph. In addition, students used data generated by measuring lengths with halves and fourths of an inch to create a line plot.

Current Learning

In fourth grade, students use a line plot to display a data set of measurement in fractions of a unit. They solve problems involving addition and subtraction of fractions by using information presented in line plots

Future Learning

Students will be making line plots to display a set of measurements in fractions of a unit (1/2, 1/4, 1/8). They will be using operations on fractions for this grade to solve problems involving information presented in line plots.

Additional Findings

Students should become familiar with a variety of representations such as tables, line plots, bar graphs, and line graphs by creating them. (*Principles and Standards for School Mathematics*, p. 178)

Grade 4 Mathematics, Quarter 4, Unit 4.3

Using Algebraic Thinking to Solve Multistep Problems with All Operations

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Solve multistep word problems with whole numbers.
- Solve problems which involve interpreting remainders.
- Use an equation with a letter standing for an unknown to represent the problem.
- Use mental computation and estimation strategies to assess reasonableness of answers.
- Add and subtract multi-digit numbers fluently using the standard algorithm.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Look for entry points to solutions.
- Plan a solution pathway.
- Monitor and evaluate their progress.

Reason abstractly and quantitatively.

- Know and flexibly use different properties of operations and objects.

Attend to precision.

- Label for clarification.
- Calculate accurately by rechecking for precision.
- Communicate formulated explanations with precision.

Essential questions

- How do you know your answer is reasonable?
- How do you plan and organize your response to a multi-step problem?
- How do you know what operation(s) to use?
- How do you decide what to do with a remainder?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Number and Operations in Base Ten[2] 4.NBT

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In third grade, students solved two-step word problems using equations with a letter standing for the unknown quantity. They learned to assess the reasonableness of their answers using mental computation and estimation. In addition, students fluently added and subtracted within 1000 using strategies and algorithms based on place value, properties of operations and the relationship between addition and subtraction.

Current Learning

According to the PARCC Model Content Frameworks for Mathematics, fluently adding and subtracting multi-digit whole numbers using the standard algorithm is an expectation by the end of fourth grade. Using the four operations to solve problems is a major cluster according to the PARCC Model Content Frameworks. It is listed in the first critical area in the CCSS. In fourth grade, students solve multistep problems with whole numbers using all four operations including problems in which remainders must be interpreted. They will represent problems using equations with a letter standing for the unknown quantity. They will use mental computation and estimation to assess. Multistep problems are limited to no more than three steps concentrating on easy and medium addition and subtraction problems, but it can include multiplication and division, too.

Future Learning

Students will solve real world problems involving fractions. All four operations will be used. It will be expected that students have already attained fluency in addition and subtraction using the standard algorithm. They will also fluently multiply multi-digit numbers using the standard algorithm.

Additional Findings

Principles and Standards for School Mathematics states that, in grades 3–5, students' knowledge and use of variables begins at an emergent stage. Throughout their work with algebraic expressions, their knowledge will develop more fully, and they should be asked to represent their thinking (p. 161).

Additionally, *Principles* states that the notion of equality should be developed throughout the curriculum. As a consequence of the instruction they receive, young students typically perceive the equals sign operationally, that is, as a signal to "do something." They should come to view the equals sign as a symbol of equivalence and balance (p. 39).

Students in grades 3-5 should have frequent experiences with problems that interest, challenge, and engage them in thinking about important mathematics. Problem solving is not a distinct topic, but a process that should permeate the study of mathematics and provide a context in which concepts and skills are learned. *Principles and Standards for School Mathematics*, p. 182.